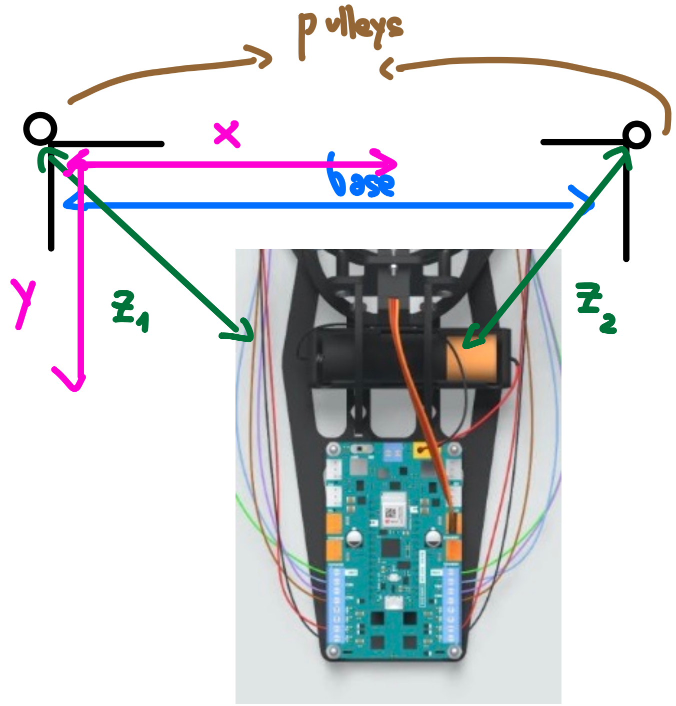
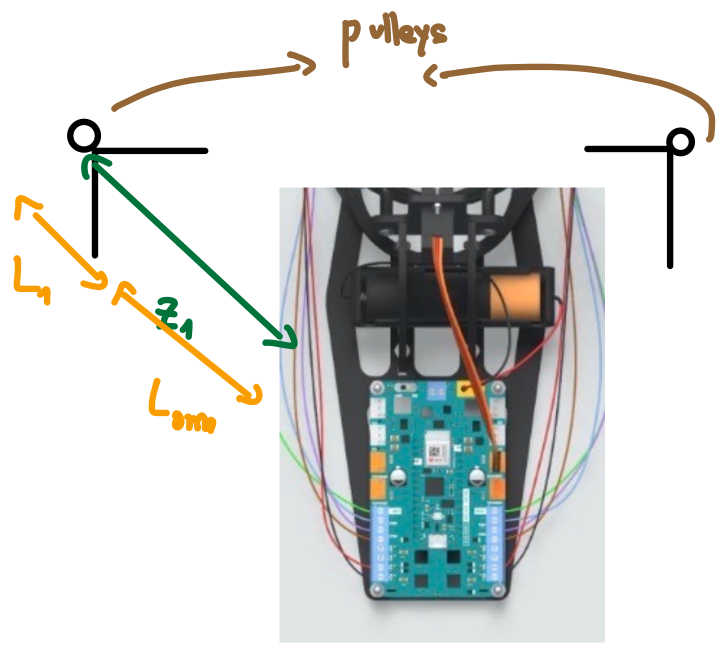
1) Define distances on the whiteboard



- We can divide z_1 in L_{arm} (which is known) and L_1 (the string length from the motor to the pulley):



- Now we use the pythagorean theorem:

$$\begin{cases} z_1^2 = x^2 + y^2 \\ z_2^2 = (Base - x)^2 + y^2 \end{cases} \iff \begin{cases} z_1^2 = x^2 + z_2^2 - (Base - x)^2 \\ y^2 = z_2^2 - (Base - x)^2 \end{cases}$$

Expanding the first equation and resolve for x:

$$z_1^2 = \cancel{x^2} + z_2^2 - Base^2 - \cancel{x^2} + 2x\,Base$$

$$\boxed{x = \frac{z_1^2 - z_2^2 + Base^2}{2 \cdot Base}} \qquad \boxed{y = \sqrt{z_1^2 - x^2}}$$

How to compute the new robot position based on the encoder readings

- We know the motor positions in units of counts. First, we can convert into an angular position:

$$C_{rad} = \frac{C_{rev}}{2\pi}$$

and:

$$\theta = \frac{C_{rad}}{counts}$$

- The amount of string spooled by the motors (ΔL_{string}) is related to the angle it has rotated and the radius of the spool:

$$\Delta L_{string} = r_{spool} \cdot \theta$$

- On the robot, the string loops over the pulley and then back to the robot body. So:

$$\Delta z_{(1 \text{ or } 2)} = \frac{\Delta L_{string\,(1 \text{ or } 2)}}{2}$$

- Now we use the x-y coordinates formulas:

$$x = \frac{z_1^2 + Base^2 - z_2^2}{2 \cdot Base}$$

$$y = \sqrt{z_1^2 - x^2}$$

DC motor torque equation

supply voltage

$$\tau = \frac{V - \omega \cdot K}{R} \cdot K$$

angular speed

torque

winding resistance

constant

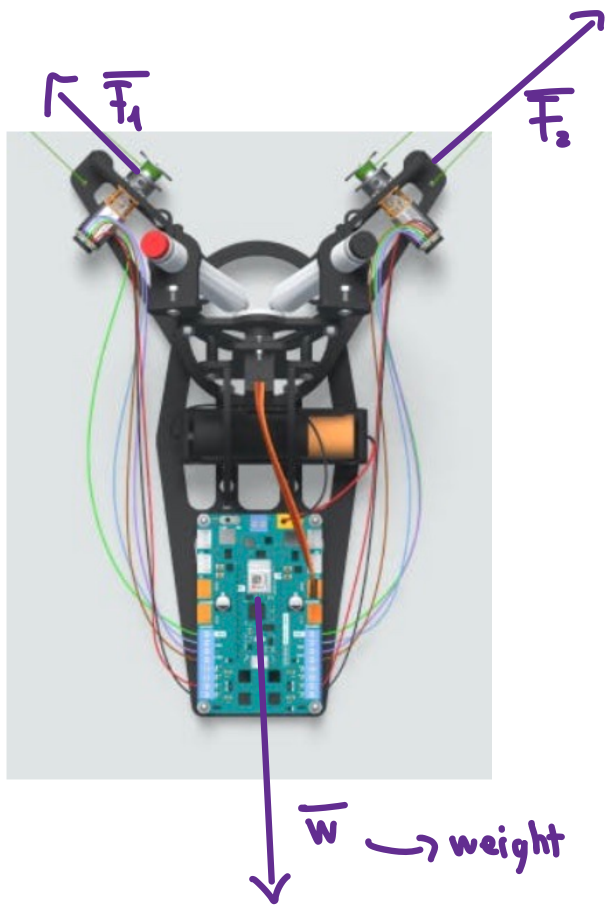
- When there is no load $(\tau = 0)$:

$$V = \omega_{free} \cdot K$$

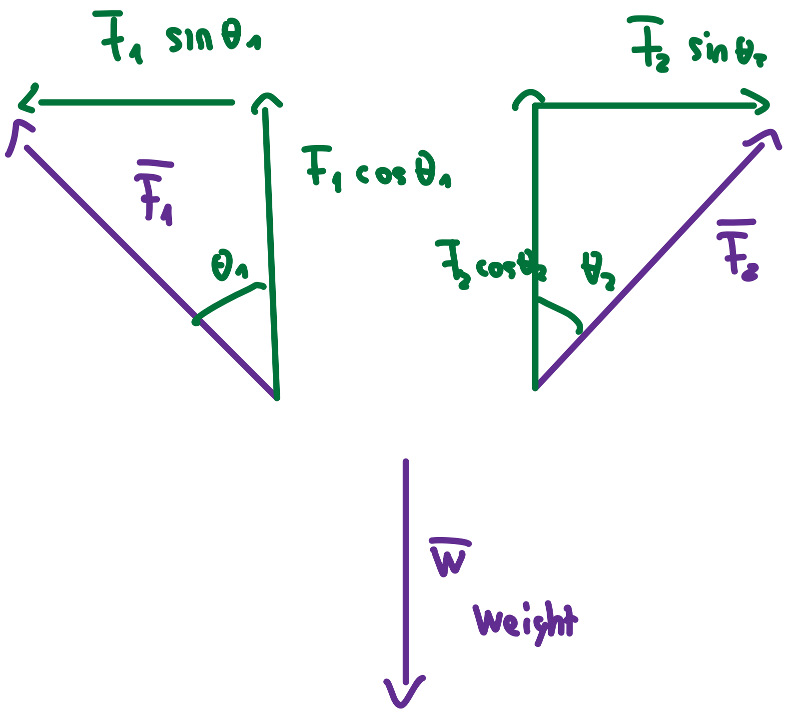
- When the motor reaches stall conditions $(\omega = 0)$:

$$\tau_{stall} = \frac{V \cdot K}{R}$$

Free body diagram



- Using trigonometry, we break down $\overline{F}_1$ and $\overline{F}_2$ into its x and y components:



$$\begin{cases} -F_1 \sin\theta_1 + F_2 \sin\theta_2 = 0 \\ F_1 \cos\theta_1 + F_2 \cos\theta_2 - W = 0 \end{cases} \iff \begin{cases} F_1 = W \cdot \dfrac{\sin\theta_2}{\sin\theta_1 \cos\theta_2 + \sin\theta_2 \cdot \cos\theta_1} \\ F_2 = W \cdot \dfrac{\sin\theta_1}{\sin\theta_1 \cdot \cos\theta_2 + \sin\theta_2 \cdot \cos\theta_1} \end{cases}$$

Equations to compute torque from $x-y$ position

- Given an $x-y$ position it is possible to compute the torque requirements. First, we can compute θ_1 and θ_2:

$$\theta_1 = \operatorname{arctg}\left(\frac{x}{y}\right)$$

$$\theta_2 = \operatorname{arctg}\left(\frac{Base - x}{y}\right)$$

$$\left\{\begin{array}{l} F_1 = W \cdot \dfrac{\sin\theta_2}{\sin\theta_1 \cos\theta_2 + \sin\theta_2 \cdot \cos\theta_1} \\[2ex] F_2 = W \cdot \dfrac{\sin\theta_1}{\sin\theta_1 \cdot \cos\theta_2 + \sin\theta_2 \cdot \cos\theta_1} \end{array}\right.$$

Then:

$$T_1 = \frac{F_1}{2}$$

$$T_2 = \frac{F_2}{2}$$

- The magnitude of the torque is :

$$\begin{cases} \tau_1 = T_1 \cdot r_{spool} \\ \tau_2 = T_2 \cdot r_{spool} \end{cases}$$